Exam B (Part II) **Name**

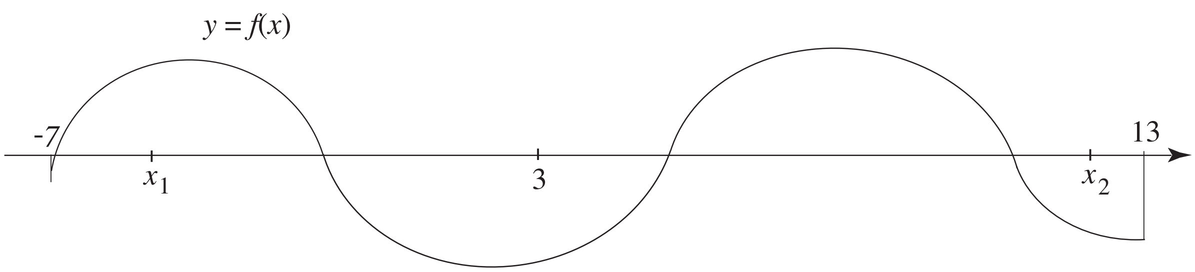
Important: Show your work in the spaces provided. (Answers alone are not enough.) For maximum credit present your solutions in a well organized, neat, and legible way. Calculators may be used *only* in elementary computational mode and trig mode, but not in calculus mode (*even for explorations*).

1. Let $y = f(x) = x^4 + 8x^3 + 10x^2$. Find all the critical numbers for this function and determine the intervals over which its graph is increasing or decreasing.

critical numbers:

intervals of increase: and decrease:

2. The figure below shows the graph of a continuous function $y = f(x)$ defined on the interval $[-7, 13]$. This function has exactly one antiderivative $y = F(x)$ that satisfies $F(3) = 0$. Define this function $y = F(x)$. To do so consider the two typical points x_1 and x_2 and use the figure to explain what $F(x_1)$ and $F(x_2)$ are equal to.



$F(x_1) =$ $F(x_2) =$

3. The sum $(1)^{\frac{1}{3}} \cdot \frac{1}{1000} + \left(1+\frac{1}{1000}\right)^{\frac{1}{3}} \frac{1}{1000} + \left(1+\frac{2}{1000}\right)^{\frac{1}{3}} \frac{1}{1000} + \cdots + \left(7+\frac{999}{1000}\right)^{\frac{1}{3}} \frac{1}{1000}$ follows the pattern that the first three terms and the last term establish. Consider the sum involved in the definition of $\int_a^b f(x)\,dx$. For what n, $a = x_0, x_1, \ldots, x_{n-1}, x_n = b$, and $y = f(x)$ is the sum above of this form? Find a number that closely approximates the value of the sum above.

the number =

4. Evaluate the integral $\int_{-1}^{2} (-x^3 + \cos(\pi x))\,dx$.

$\int_{-1}^{2} (-x^3 + \cos(\pi x))\,dx =$

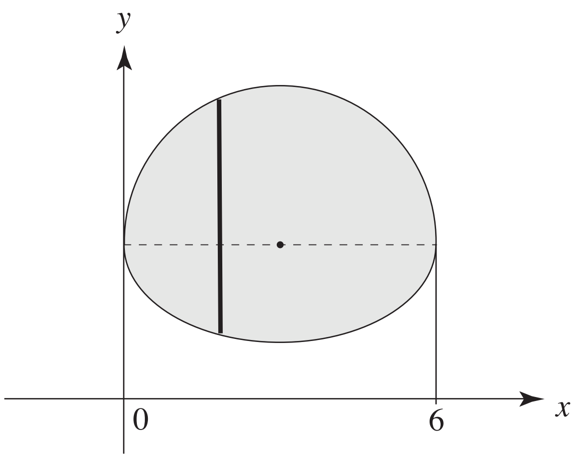
5. Consider the definite integral $\int_{1}^{3} \sqrt{1+x}\,dx$.

i. For what function $f(x)$ does this definite integral represent an area under the graph of $f(x)$.

ii. Find a function $g(x)$ with the property that this definite integral represents the length graph of $f(x)$.

iii. Find a function $h(x)$ with the property that the integral represents the volume obtained by rotating a region under the graph of $h(x)$ one revolution about the x-axis; and

6. The figure below shows the upper half of a circle of radius 3 and the lower half of an ellipse with semimajor axis 3 and semiminor axis 2. Both the circle and the ellipse have center $(3, 3)$.



a. Find the area between the circle and the ellipse (the area of the shaded region). [Best without integrals.]

area =

b. Express the volume obtained by revolving the shaded region one revolution around the x-axis as a definite integral.

$V_x = \int$

c. Express the volume obtained by revolving the shaded region one revolution around the y-axis as a definite integral.

$V_y = \int$

d. Why is the vertical strip a better choice than a horizontal strip for the above two problems?

e. Express the surface area S of the solid obtained by rotating the shaded region one revolution around the x-axis as a sume of two definite integrals.

$S = \int \qquad\qquad + \int$

7. Consider the hyperbolic functions $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

a. Show that $(\cosh x)^2 - (\sinh x)^2 = 1$.

b. Show that the derivative of $\sinh x$ is $\cosh x$, and that the derivative of $\cosh x$ is $\sinh x$.

c. Compute the length of the graph of $y = \cosh x$ between the points with x-coordinates 0 and $\ln 7$.

this length is:

8. The Golden Gate Bridge has a center span of 4,200 feet and a total length of 6,450 feet. It has two main cables, one deck, a dead load of 21,300 pounds per foot, and a live load capacity of 4,000 pounds per foot. The towers have a height of 746 feet and the sag in each of the two main cables is 470 feet.

i. Compute the maximal and minimal tensions in each main cable over the center span.

ii. Compute the angle that the cable makes with the horizontal at the tower .

ii. Compute the compression that a cable over the center span produces in the tower.

Formulas and expressions:

$$\int u\,dv = uv - \int v\,du \qquad Ay'' + By' + Cy = 0 \qquad y = D_1 e^{r_1 x} + D_2 e^{r_2 x} \qquad y = D_1 e^{2x} + D_2 x e^{2x}$$

$$y = e^{ax}(D_1 \cos bx + D_2 \sin bx) \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$a = \tfrac{d}{1-\varepsilon^2} \qquad b = \tfrac{d}{\sqrt{1-\varepsilon^2}} \qquad a = \tfrac{d}{\varepsilon^2-1} \qquad b = \tfrac{d}{\sqrt{\varepsilon^2-1}} \qquad f'(\theta) = f(\theta)\cdot\tan(\gamma - \tfrac{\pi}{2})$$

$$f(x) = \tfrac{s}{d^2}x^2, \quad \tan\alpha = \tfrac{2s}{d}, \quad T(x) = w\sqrt{\left(\tfrac{d^2}{2s}\right)^2 + x^2}, \quad T_d = wd\sqrt{\left(\tfrac{d}{2s}\right)^2 + 1}, \quad T_0 = \tfrac{wd^2}{2s}.$$

$$ab\pi \qquad a^2 = b^2 + c^2 \qquad \tfrac{x^2}{a^2} + \tfrac{y^2}{b^2} = 1 \quad \kappa = \tfrac{ab\pi}{T} \qquad F = ma \quad f(t)\cdot r = I\cdot\alpha(t) \qquad \tfrac{d}{dx}\sin x = \cos x$$

$$\int_a^b \sqrt{1+f'(x)^2}\,dx \qquad \int_a^b \pi f(x)^2\,dx \qquad 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$